

EJERCICIOS RESUELTOS SOBRE LÍMITES INDETERMINADOS



LIMITES INDETERMINADOS

Los siguientes ejercicios sobre límites están indeterminados, es decir al aplicar el valor en la función se obtiene $\frac{0}{0}$; para levantar la indeterminación se debe descomponer en factores y luego volver a evaluar para el valor dado.

<p>1. $\lim_{x \rightarrow 0} \frac{4x^3 - 2x^2 + x}{3x^2 + 2x}$</p>	$\lim_{x \rightarrow 0} \frac{4x^3 - 2x^2 + x}{3x^2 + 2x} = \frac{4(0)^3 - 2(0)^2 + 0}{3(0)^2 + 2(0)} = \frac{0}{0} \text{ Indeterminación.}$ $\lim_{x \rightarrow 0} \frac{4x^3 - 2x^2 + x}{3x^2 + 2x} = \lim_{x \rightarrow 0} \frac{x(4x^2 - 2x + 1)}{x(3x + 2)}$ $= \lim_{x \rightarrow 0} \frac{4x^2 - 2x + 1}{3x + 2} = \frac{4(0)^2 - 2(0) + 1}{3(0) + 2} = \frac{1}{2}$
<p>2. $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$</p>	$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = \frac{(2)^2 - 4}{2 - 2} = \frac{0}{0} \text{ Indeterminación.}$ $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = \lim_{x \rightarrow 2} \frac{(x + 2)(x - 2)}{x - 2}$ $\lim_{x \rightarrow 2} x + 2 = 2 + 2 = 4$
<p>3. $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1}$</p>	$\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1} = \frac{(1)^3 - 1}{1 - 1} = \frac{0}{0} \text{ Indeterminación}$ $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1} = \lim_{x \rightarrow 1} \frac{(x - 1)(x^2 + x + 1)}{x - 1}$ $\lim_{x \rightarrow 1} (x^2 + x + 1) = (1)^2 + 1 + 1 = 3$
<p>4. $\lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x^2 - 12x + 20}$</p>	$\lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x^2 - 12x + 20} = \frac{(2)^2 - 5(2) + 6}{(2)^2 - 12(2) + 20} = \frac{4 - 10 + 6}{4 - 24 + 20} = \frac{0}{0} \text{ Indet.}$ $\lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x^2 - 12x + 20} = \lim_{x \rightarrow 2} \frac{(x - 3)(x - 2)}{(x - 10)(x - 2)}$ $\lim_{x \rightarrow 2} \frac{x - 3}{x - 10} = \frac{2 - 3}{2 - 10} = \frac{-1}{-8} = \frac{1}{8}$

<p>5. $\lim_{x \rightarrow 2} \frac{x^2 + 3x - 10}{3x^2 - 5x - 2}$</p>	$\lim_{x \rightarrow 2} \frac{x^2 + 3x - 10}{3x^2 - 5x - 2} = \frac{(2)^2 + 3(2) - 10}{3(2)^2 - 5(2) - 2} = \frac{4 + 6 - 10}{12 - 10 - 2} = \frac{0}{0}$ <p>Indeterminación</p> $\lim_{x \rightarrow 2} \frac{x^2 + 3x - 10}{3x^2 - 5x - 2} = \lim_{x \rightarrow 2} \frac{(x+5)(x-2)}{(x-2)(3x+1)}$ $\lim_{x \rightarrow 2} \frac{(x+5)}{(3x+1)} = \frac{2+5}{3(2)+1} = \frac{7}{7} = 1 \blacksquare$
<p>6. $\lim_{y \rightarrow -2} \frac{y^3 + 3y^2 + 2y}{y^2 - y - 6}$</p>	$\lim_{y \rightarrow -2} \frac{y^3 + 3y^2 + 2y}{y^2 - y - 6} = \lim_{y \rightarrow -2} \frac{y(y^2 + 3y + 2)}{(y-3)(y+2)} = \lim_{y \rightarrow -2} \frac{y(y+2)(y+1)}{(y-3)(y+2)}$ $\lim_{y \rightarrow -2} \frac{y(y+1)}{(y-3)} = \frac{-2(-2+1)}{-2-3} = \frac{2}{-5} = -\frac{2}{5} \blacksquare$
<p>7. $\lim_{u \rightarrow -2} \frac{u^3 + 4u^2 + 4u}{(u+2)(u-3)}$</p>	$\lim_{u \rightarrow -2} \frac{u^3 + 4u^2 + 4u}{(u+2)(u-3)} = \frac{(-2)^3 + 4(2)^2 + 4(-2)}{(-2+2)(-2-3)} = \frac{-8 + 16 - 8}{0(-5)} = \frac{0}{0}$ $\lim_{u \rightarrow -2} \frac{u^3 + 4u^2 + 4u}{(u+2)(u-3)} = \lim_{u \rightarrow -2} \frac{u(u^2 + 4u + 4)}{(u+2)(u-3)} = \lim_{u \rightarrow -2} \frac{u(u+2)^2}{(u+2)(u-3)}$ $\lim_{u \rightarrow -2} \frac{u(u+2)}{(u-3)} = \frac{-2(-2+2)}{-2-3} = \frac{0}{-5} = 0 \blacksquare$
<p>8. $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 - 3x + 2}$</p>	$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 - 3x + 2} = \frac{(2)^2 - 4}{(2)^2 - 3(2) + 2} = \frac{0}{0}$ <p>Indeterminación.</p> $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 - 3x + 2} = \lim_{x \rightarrow 2} \frac{(x+2)(x-2)}{(x-2)(x-1)}$ $\lim_{x \rightarrow 2} \frac{(x+2)}{(x-1)} = \frac{2+2}{2-1} = \frac{4}{1} = 4 \blacksquare$
<p>9. $\lim_{x \rightarrow 5} \frac{x^2 - 7x + 10}{x^2 - 25}$</p>	$\lim_{x \rightarrow 5} \frac{x^2 - 7x + 10}{x^2 - 25} = \frac{(5)^2 - 7(5) + 10}{(5)^2 - 25} = \frac{25 - 35 + 10}{25 - 25} = \frac{0}{0}$ <p>Indeterminación.</p> $\lim_{x \rightarrow 5} \frac{x^2 - 7x + 10}{x^2 - 25} = \lim_{x \rightarrow 5} \frac{(x-5)(x-2)}{(x+5)(x-5)}$ $\lim_{x \rightarrow 5} \frac{(x-2)}{(x+5)} = \frac{5-2}{5+5} = \frac{3}{10} \blacksquare$

10. $\lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h}$	$\lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h} = \frac{(x+0)^3 - x^3}{0} = \frac{x^3 - x^3}{0} = \frac{0}{0} \text{ Indeterminación.}$ $\lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h \rightarrow 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} = \lim_{h \rightarrow 0} \frac{h(3x^2 + 3xh + h^2)}{h}$ $\lim_{h \rightarrow 0} (3x^2 + 3xh + h^2) = 3x^2 + 3x(0) + (0)^2 = 3x^2 \blacksquare$
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Los siguientes 8 ejercicios también son indeterminados, la presencia de radicales obliga a levantar la Indeterminación mediante la racionalización de los radicales aplicando el producto conjugado.

11. $\lim_{x \rightarrow 0} \frac{\sqrt{1+x}-1}{x}$	$\lim_{x \rightarrow 0} \frac{\sqrt{1+x}-1}{x} = \frac{\sqrt{1+0}-1}{0} = \frac{0}{0} \text{ Indeterminado}$ $\lim_{x \rightarrow 0} \frac{\sqrt{1+x}-1}{x} = \lim_{x \rightarrow 0} \frac{\sqrt{1+x}-1}{x} \times \frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}$ $\lim_{x \rightarrow 0} \frac{(\sqrt{1+x}-1)(\sqrt{1+x}+1)}{x(\sqrt{1+x}+1)} = \lim_{x \rightarrow 0} \frac{(\sqrt{1+x})^2 - 1}{x(\sqrt{1+x}+1)} = \lim_{x \rightarrow 0} \frac{1+x-1}{x(\sqrt{1+x}+1)}$ $\lim_{x \rightarrow 0} \frac{x}{x(\sqrt{1+x}+1)} = \lim_{x \rightarrow 0} \frac{1}{\sqrt{1+x}+1} = \frac{1}{\sqrt{1+0}+1} = \frac{1}{2} \blacksquare$
12. $\lim_{x \rightarrow 7} \frac{2-\sqrt{x-3}}{x^2-49}$	$\lim_{x \rightarrow 7} \frac{2-\sqrt{x-3}}{x^2-49} = \frac{2-\sqrt{7-3}}{(7)^2-49} = \frac{2-\sqrt{4}}{49-49} = \frac{2-2}{0} = \frac{0}{0} \text{ Indeterminado}$ $\lim_{x \rightarrow 7} \frac{2-\sqrt{x-3}}{x^2-49} = \lim_{x \rightarrow 7} \frac{2-\sqrt{x-3}}{x^2-49} \times \frac{2+\sqrt{x-3}}{2+\sqrt{x-3}}$ $\lim_{x \rightarrow 7} \frac{(2-\sqrt{x-3})(2+\sqrt{x-3})}{(x^2-49)(2+\sqrt{x-3})} = \lim_{x \rightarrow 7} \frac{(2)^2 - (\sqrt{x-3})^2}{(x+7)(x-7)(2+\sqrt{x-3})}$ $\lim_{x \rightarrow 7} \frac{4-(x-3)}{(x+7)(x-7)(2+\sqrt{x-3})} = \lim_{x \rightarrow 7} \frac{4-x+3}{(x+7)(x-7)(2+\sqrt{x-3})}$ $\lim_{x \rightarrow 7} \frac{7-x}{(x+7)(x-7)(2+\sqrt{x-3})} = \lim_{x \rightarrow 7} \frac{-(x-7)}{(x+7)(x-7)(2+\sqrt{x-3})}$ $\lim_{x \rightarrow 7} \frac{-1}{(x+7)(2+\sqrt{x-3})} = \frac{-1}{(7+7)(2+\sqrt{7-3})} = \frac{-1}{14(2+\sqrt{4})} = \frac{-1}{56} \blacksquare$

13. $\lim_{x \rightarrow 0} \frac{\sqrt{1+x+x^2}-1}{x}$

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x+x^2}-1}{x} = \frac{\sqrt{1+0+(0)^2}-1}{0} = \frac{0}{0} \text{ Indeterminado}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x+x^2}-1}{x} = \lim_{x \rightarrow 0} \frac{\sqrt{1+x+x^2}-1}{x} \times \frac{\sqrt{1+x+x^2}+1}{\sqrt{1+x+x^2}+1}$$

$$\lim_{x \rightarrow 0} \frac{(\sqrt{1+x+x^2}-1)(\sqrt{1+x+x^2}+1)}{x(\sqrt{1+x+x^2}+1)} = \lim_{x \rightarrow 0} \frac{(\sqrt{1+x+x^2})^2-1}{x(\sqrt{1+x+x^2}+1)} = \lim_{x \rightarrow 0} \frac{1+x+x^2-1}{x(\sqrt{1+x+x^2}+1)}$$

$$\lim_{x \rightarrow 0} \frac{x+x^2}{x(\sqrt{1+x+x^2}+1)} = \lim_{x \rightarrow 0} \frac{x(1+x)}{x(\sqrt{1+x+x^2}+1)}$$

$$\lim_{x \rightarrow 0} \frac{1+x}{\sqrt{1+x+x^2}+1} = \frac{1+0}{\sqrt{1+0+(0)^2}+1} = \frac{1}{\sqrt{1}+1} = \frac{1}{2} \blacksquare$$

14. $\lim_{x \rightarrow 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{x}$

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{x} = \frac{\sqrt{1+0}-\sqrt{1-0}}{0} = \frac{\sqrt{1}-\sqrt{1}}{0} = \frac{0}{0} \text{ Indeterminado.}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{x} = \lim_{x \rightarrow 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{x} \times \frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$$

$$\lim_{x \rightarrow 0} \frac{(\sqrt{1+x}-\sqrt{1-x})(\sqrt{1+x}+\sqrt{1-x})}{x(\sqrt{1+x}+\sqrt{1-x})} = \lim_{x \rightarrow 0} \frac{(\sqrt{1+x})^2-(\sqrt{1-x})^2}{x(\sqrt{1+x}+\sqrt{1-x})}$$

$$\lim_{x \rightarrow 0} \frac{(\sqrt{1+x})^2-(\sqrt{1-x})^2}{x(\sqrt{1+x}+\sqrt{1-x})} = \lim_{x \rightarrow 0} \frac{1+x-(1-x)}{x(\sqrt{1+x}+\sqrt{1-x})}$$

$$\lim_{x \rightarrow 0} \frac{1+x-1+x}{x(\sqrt{1+x}+\sqrt{1-x})} = \lim_{x \rightarrow 0} \frac{2x}{x(\sqrt{1+x}+\sqrt{1-x})}$$

$$\lim_{x \rightarrow 0} \frac{2}{\sqrt{1+x}+\sqrt{1-x}} = \frac{2}{\sqrt{1+0}+\sqrt{1-0}} = \frac{2}{2\sqrt{1}} = 1 \blacksquare$$

$$15. \lim_{x \rightarrow 3} \frac{\sqrt{x^2 - 2x + 6} - \sqrt{x^2 + 2x - 6}}{x^2 - 4x + 3}$$

$$\lim_{x \rightarrow 3} \frac{\sqrt{x^2 - 2x + 6} - \sqrt{x^2 + 2x - 6}}{x^2 - 4x + 3} = \frac{\sqrt{(3)^2 - 2(3) + 6} - \sqrt{(3)^2 + 2(3) - 6}}{(3)^2 - 4(3) + 3} = \frac{\sqrt{9 - 6 + 6} - \sqrt{9 + 6 - 6}}{9 - 12 + 3} = \frac{0}{0}$$

$$\lim_{x \rightarrow 3} \frac{\sqrt{x^2 - 2x + 6} - \sqrt{x^2 + 2x - 6}}{x^2 - 4x + 3} \times \frac{\sqrt{x^2 - 2x + 6} + \sqrt{x^2 + 2x - 6}}{\sqrt{x^2 - 2x + 6} + \sqrt{x^2 + 2x - 6}}$$

$$\lim_{x \rightarrow 3} \frac{(\sqrt{x^2 - 2x + 6} - \sqrt{x^2 + 2x - 6})(\sqrt{x^2 - 2x + 6} + \sqrt{x^2 + 2x - 6})}{(x^2 - 4x + 3)(\sqrt{x^2 - 2x + 6} + \sqrt{x^2 + 2x - 6})}$$

$$\lim_{x \rightarrow 3} \frac{(\sqrt{x^2 - 2x + 6})^2 - (\sqrt{x^2 + 2x - 6})^2}{(x^2 - 4x + 3)(\sqrt{x^2 - 2x + 6} + \sqrt{x^2 + 2x - 6})} = \frac{x^2 - 2x + 6 - (x^2 + 2x - 6)}{(x^2 - 4x + 3)(\sqrt{x^2 - 2x + 6} + \sqrt{x^2 + 2x - 6})}$$

$$\lim_{x \rightarrow 3} \frac{x^2 - 2x + 6 - x^2 - 2x + 6}{(x - 3)(x - 1)(\sqrt{x^2 - 2x + 6} + \sqrt{x^2 + 2x - 6})} = \lim_{x \rightarrow 3} \frac{-4x + 12}{(x - 3)(x - 1)(\sqrt{x^2 - 2x + 6} + \sqrt{x^2 + 2x - 6})}$$

$$\lim_{x \rightarrow 3} \frac{-4(x - 3)}{(x - 3)(x - 1)(\sqrt{x^2 - 2x + 6} + \sqrt{x^2 + 2x - 6})} = \lim_{x \rightarrow 3} \frac{-4}{(x - 1)(\sqrt{x^2 - 2x + 6} + \sqrt{x^2 + 2x - 6})}$$

$$= \frac{-4}{(3 - 1)(\sqrt{(3)^2 - 2(3) + 6} + \sqrt{(3)^2 + 2(3) - 6})} = \frac{-4}{2(\sqrt{9} + \sqrt{9})} = \frac{-4}{2(6)} = -\frac{1}{3}$$

$$16. \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \frac{\sqrt{x+0} - \sqrt{x}}{0} = \frac{0}{0} \text{ Indeterminado}$$

$$\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \times \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$\lim_{h \rightarrow 0} \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \rightarrow 0} \frac{(\sqrt{x+h})^2 - (\sqrt{x})^2}{h(\sqrt{x+h} + \sqrt{x})}$$

$$\lim_{h \rightarrow 0} \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \rightarrow 0} \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})}$$

$$\lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{\sqrt{x+0} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

17. $\lim_{x \rightarrow 4} \frac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{2}}$

$$\lim_{x \rightarrow 4} \frac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{2}} = \frac{\sqrt{2(4)+1}-3}{\sqrt{4-2}-\sqrt{2}} = \frac{\sqrt{9}-3}{\sqrt{2}-\sqrt{2}} = \frac{0}{0} \text{ Indeterminado}$$

La presencia de radicales en el numerador y en el denominador obliga a la formación de dos productos conjugados:

$$\begin{aligned} \lim_{x \rightarrow 4} \frac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{2}} &= \lim_{x \rightarrow 4} \frac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{2}} \times \frac{\sqrt{2x+1}+3}{\sqrt{2x+1}+3} \times \frac{\sqrt{x-2}+\sqrt{2}}{\sqrt{x-2}+\sqrt{2}} \\ \lim_{x \rightarrow 4} \frac{(\sqrt{2x+1}-3)(\sqrt{2x+1}+3)(\sqrt{x-2}+\sqrt{2})}{(\sqrt{x-2}-\sqrt{2})(\sqrt{x-2}+\sqrt{2})(\sqrt{2x+1}+3)} &= \lim_{x \rightarrow 4} \frac{[(\sqrt{2x+1})^2-9](\sqrt{x-2}+\sqrt{2})}{[(\sqrt{x-2})^2-(\sqrt{2})^2](\sqrt{2x+1}+3)} \\ \lim_{x \rightarrow 4} \frac{[2x+1-9](\sqrt{x-2}+\sqrt{2})}{[x-2-2](\sqrt{2x+1}+3)} &= \lim_{x \rightarrow 4} \frac{[2x-8](\sqrt{x-2}+\sqrt{2})}{[x-4](\sqrt{2x+1}+3)} = \lim_{x \rightarrow 4} \frac{2(x-4)(\sqrt{x-2}+\sqrt{2})}{(x-4)(\sqrt{2x+1}+3)} \\ \lim_{x \rightarrow 4} \frac{2(\sqrt{x-2}+\sqrt{2})}{(\sqrt{2x+1}+3)} &= \frac{2(\sqrt{4-2}+\sqrt{2})}{\sqrt{2(4)+1}+3} = \frac{2(\sqrt{2}+\sqrt{2})}{\sqrt{9}+3} = \frac{2(2\sqrt{2})}{3+3} = \frac{4\sqrt{2}}{6} = \frac{2\sqrt{2}}{3} \blacksquare \end{aligned}$$

18. $\lim_{x \rightarrow 0} \frac{\sqrt{x^2+p^2}-p}{\sqrt{x^2+q^2}-q}$

$$\lim_{x \rightarrow 0} \frac{\sqrt{x^2+p^2}-p}{\sqrt{x^2+q^2}-q} = \frac{\sqrt{(0)^2+p^2}-p}{\sqrt{(0)^2+q^2}-q} = \frac{p-p}{q-q} = \frac{0}{0} \text{ Indeterminado}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{x^2+p^2}-p}{\sqrt{x^2+q^2}-q} = \lim_{x \rightarrow 0} \frac{\sqrt{x^2+p^2}-p}{\sqrt{x^2+q^2}-q} \times \frac{\sqrt{x^2+p^2}+p}{\sqrt{x^2+p^2}+p} \times \frac{\sqrt{x^2+q^2}+q}{\sqrt{x^2+q^2}+q}$$

$$\lim_{x \rightarrow 0} \frac{(\sqrt{x^2+p^2}-p)(\sqrt{x^2+p^2}+p)(\sqrt{x^2+q^2}+q)}{(\sqrt{x^2+q^2}-q)(\sqrt{x^2+q^2}+q)(\sqrt{x^2+p^2}+p)}$$

$$\lim_{x \rightarrow 0} \frac{[(\sqrt{x^2+p^2})^2-p^2](\sqrt{x^2+q^2}+q)}{[(\sqrt{x^2+q^2})^2-q^2](\sqrt{x^2+p^2}+p)} = \lim_{x \rightarrow 0} \frac{[x^2+p^2-p^2](\sqrt{x^2+q^2}+q)}{[x^2+q^2-q^2](\sqrt{x^2+p^2}+p)}$$

$$\lim_{x \rightarrow 0} \frac{x^2(\sqrt{x^2+q^2}+q)}{x^2(\sqrt{x^2+p^2}+p)} = \lim_{x \rightarrow 0} \frac{\sqrt{x^2+q^2}+q}{\sqrt{x^2+p^2}+p} = \frac{\sqrt{(0)^2+q^2}+q}{\sqrt{(0)^2+p^2}+p} = \frac{\sqrt{q^2}+q}{\sqrt{p^2}+p} = \frac{2q}{2p} = \frac{q}{p} \blacksquare$$

Los 2 siguientes ejercicios están indeterminados en primera instancia, la presencia de radicales cúbicos obliga a buscar un factor racionalizante que permita levantar dicha indeterminación; recuerde que:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

19. $\lim_{x \rightarrow 8} \frac{x-8}{\sqrt[3]{x}-2}$

$$\lim_{x \rightarrow 8} \frac{x-8}{\sqrt[3]{x}-2} = \frac{8-8}{\sqrt[3]{8}-2} = \frac{0}{0} \text{ Est\aa indeterminado.}$$

$$\lim_{x \rightarrow 8} \frac{x-8}{\sqrt[3]{x}-2} = \lim_{x \rightarrow 8} \frac{(x^{\ddagger})^3 - (2)^3}{x^{\ddagger} - 2} = \lim_{x \rightarrow 8} \frac{(x^{\ddagger} - 2)(x^{\ddagger} + 2x^{\ddagger} + 4)}{(x^{\ddagger} - 2)}$$

$$\lim_{x \rightarrow 8} (x^{\ddagger} + 2x^{\ddagger} + 4) = \lim_{x \rightarrow 8} (\sqrt[3]{x^2} + 2\sqrt[3]{x} + 4) = \sqrt[3]{64} + 2(\sqrt[3]{8}) + 4 = 4 + 4 + 4 = 12 \blacksquare$$

20. $\lim_{x \rightarrow 1} \frac{\sqrt[3]{x}-1}{\sqrt{x}-1}$

$$\lim_{x \rightarrow 1} \frac{\sqrt[3]{x}-1}{\sqrt{x}-1} = \frac{\sqrt[3]{1}-1}{\sqrt{1}-1} = \frac{0}{0} \text{ Est\aa indeterminado.}$$

$$\lim_{x \rightarrow 1} \frac{\sqrt[3]{x}-1}{\sqrt{x}-1} = \lim_{x \rightarrow 1} \frac{x^{\ddagger}-1}{\sqrt{x}-1} \times \frac{x^{\ddagger}+x^{\ddagger}+1}{x^{\ddagger}+x^{\ddagger}+1} \times \frac{\sqrt{x}+1}{\sqrt{x}+1}$$

Reordenando los factores se tiene:

$$\lim_{x \rightarrow 1} \frac{(x^{\ddagger}-1)(x^{\ddagger}+x^{\ddagger}+1)(\sqrt{x}+1)}{(x^{\ddagger}+x^{\ddagger}+1)(\sqrt{x}-1)(\sqrt{x}+1)} = \lim_{x \rightarrow 1} \frac{(x-1)(\sqrt{x}+1)}{(x^{\ddagger}+x^{\ddagger}+1)(x-1)}$$

$$\lim_{x \rightarrow 1} \frac{\sqrt{x}+1}{x^{\ddagger}+x^{\ddagger}+1} = \frac{\sqrt{1}+1}{1+1+1} = \frac{2}{3} \blacksquare$$